Technical Comments

Comment on "Wall Layer of Plane Turbulent Wall Jets without Pressure Gradients"

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HUBBARTT and Neale[1] have shown that the "wake component," or deviation from the law of the wall, in a wall jet in still air is well represented by

$$\Delta U/u_\tau = -33.3 \ \mathrm{erf}\,(0.0652\ y/\delta_m) \qquad (1)$$

where δ_m is the value of y at which U is a maximum. The purpose of this comment is to point out that for $y << \delta_m$ Eq. (1) agrees very well with the wake component obtained by integrating the mixing length formula[2]

$$\partial U/\partial y = (\tau/\rho)^{1/2}/Ky \qquad (2)$$

with $\tau = \tau_w\ (1 - 2\ y/\delta_m)$. This variation of τ seems to be a good approximation outside the viscous sublayer; it was found in Ref. 3 and by later authors that $\tau \approx -\tau_w$ at $y = \delta_m$. The full integral of Eq. (2) is given in Ref. 2: let us merely note that the leading term for $\Delta U/u_\tau$ is $y(\partial\tau/\partial y)/(2K\tau_w)$ where K is von Kârman's constant, taken as 0.41 by Coles. Thus Eq. (2) gives

$$\Delta U/u_\tau = -2.44\,y/\delta_m \qquad (3)$$

while near the wall Eq. (1), as obtained from Fig. 3 of Ref. 1, is closely equal to

$$\Delta U/u_\tau = -2.38\ y/\delta_m \qquad (4)$$

The comparison cannot be extended to wall jets below a moving stream because $\partial\tau/\partial y$ is not known.

As usual, Eqs. (2) and (3) are expected to be valid only for y rather smaller than $\delta_m/2$ whereas the wholly-empirical form Eq. (1) appears to be valid at least as far as $y = \delta_{1/2}$ which cannot be explained by the wall-layer form of the mixing length formula. The main value of the comparison is the additional support it gives for Eq. (2) in regions of strong negative shear-stress gradient where its validity is sometimes questioned. For example, in Ref. 4, Vol. 1, p. 31, D. Coles argues that ΔU should be zero in the inner layer of a flow with $\partial\tau/\partial y < 0$, partly on the evidence of the data of Run 6300 of Ref. 4, Vol. 2, p. 504. However a typical value of $\Delta U/u_\tau$ predicted by Eq. (2) in Run 6300 is only −0.15; Hubbartt and Neale's data, with $\Delta U/u_\tau$ of the order of −2.0, provide more definite evidence, and this evidence supports Eq. (2) which is used directly or indirectly in many calculation methods for turbulent wall flows.

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References

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[4]Kline, S. J., Morkovin, M. V., Sovran, G., and Cockrell, D. J. (editors), *Proceedings on the Computation of Turbulent Boundary Layers-1968 AFOSR-IFP-Stanford Conference,* Stanford Univ., Stanford, Calif., 1969.

Comparison of Geometric and Response-Feedback Approaches to Aircraft Lateral-Directional Decoupling

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AN ARTICLE by Eugene M. Cliff and Frederick H. Lutze,[1] is critical of the response-feedback approach to the same problem presented in Ref. 2. The purpose of this article is to clarify apparent misunderstandings of the response-feedback technique and compare it to the geometric-decoupling approach for determining the desired handling qualities solution.

Reference 1 refers to the approach taken in Ref. 2 as a "model-following" approach. We feel a more correct title would be "response-feedback" approach. This is indeed a small point since definitions vary. We prefer to define a model-following system as one in which the model is explicitly programmed in the system and followed on-line by the plant or simulator vehicle in this case. In Ref. 2, a model is defined which is only implicitly followed.

The intent of the study in Ref. 2 was to decouple a T-33 by matching the responses of a model of the following form

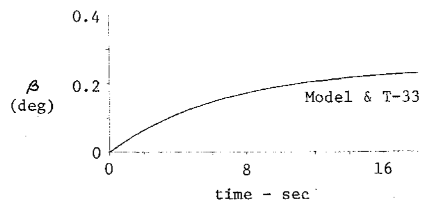

Fig. 1 β **response to step rudder command—all systems.**

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form.

$$\begin{bmatrix} \dot{p} \\ \dot{\phi} \\ \dot{r} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} L'_{p_m} & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 \\ 0 & 0 & N'_{r_m} & 0 \\ 0 & 0 & 0 & Y_{\beta_m} \end{bmatrix} \begin{bmatrix} p \\ \phi \\ r \\ \beta \end{bmatrix} + \begin{bmatrix} L'_{\delta_{a_m}} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & N'_{\delta_{p_m}} \\ 0 & Y_{\delta_{r_m}} & 0 \end{bmatrix} \begin{bmatrix} \delta_{a_c} \\ \delta_{r_c} \\ \delta_{p_c} \end{bmatrix} \tag{1}$$

where

$L'_{p_m} = -3.18$ $\quad L'_{\delta_{a_m}} = -14.4$

$N'_{r_m} = -0.27$ $\quad Y'_{\delta_{r_m}} = 0.037$

$Y_{\beta_m} = -0.151$ $\quad N'_{\delta_{p_m}} = -0.96$

δ_p = yaw control obtained by the differential deflection of drag petals

subscript c = command signal

subscript m = model parameters

Cliff and Lutze point out that due to poor accuracy, the gain values given in Ref. 2 do not lead to the desired pole placement. The reason for this is that the theoretical values of the system gains required for exact decoupling were rounded off. A T-33 in-flight simulator was to be decoupled and it is not possible to set gain pots to four decimal places nor define the parameters of the equations of motion of the T-33 that precisely. This will be true in any real world application of either the response-feedback or geometric-decoupling techniques and led to the decision to round off the gain values.

However, to form the basis of the discussion, results obtained with the geometric-decoupling and response-feedback techniques are compared with regard to the eigenvalues and resultant equations of motion of a decoupled system. Two response feedback systems are compared. One of these is a system of gains which gives theoretically perfect matching of the desired model responses while the other is the system of gains obtained by rounding off the values of gains calculated for the T-33 decoupling problem.

Using these three systems of gains leads to the following equations of motion and corresponding characteristic equations.

Response-Feedback Exact Gain System

$$\begin{bmatrix} \dot{p} \\ \dot{\phi} \\ \dot{r} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -3.180 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 \\ 0 & 0 & -0.270 & 0 \\ 0 & 0 & 0 & -0.151 \end{bmatrix} \begin{bmatrix} p \\ \phi \\ r \\ \beta \end{bmatrix} + \begin{bmatrix} -14.4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.96 \\ 0 & 0.037 & 0 \end{bmatrix} \begin{bmatrix} \delta_{a_c} \\ \delta_{r_c} \\ \delta_{p_c} \end{bmatrix} \tag{2}$$

$$\text{Characteristic equation} = s(s + 0.151)(s + 0.270)(s + 3.180) \tag{3}$$

$$\begin{bmatrix} \dot{p} \\ \dot{\phi} \\ \dot{r} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -3.180 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 \\ 0 & 0 & -0.120 & 0 \\ 0 & 0 & 0 & -0.151 \end{bmatrix} \begin{bmatrix} p \\ \phi \\ r \\ \beta \end{bmatrix} + \begin{bmatrix} -14.4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.96 \\ 0 & 0.037 & 0 \end{bmatrix} \begin{bmatrix} \delta_{a_c} \\ \delta_{r_c} \\ \delta_{p_c} \end{bmatrix} \tag{4}$$

$$\text{Characteristic equation} = s(s + 0.151)(s + 0.120)(s + 3.180) \tag{5}$$

Geometric-Decoupled System

$$\begin{bmatrix} \dot{p} \\ \dot{\phi} \\ \dot{r} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} -0.272 & 0 & 0 & 0 \\ 1.0 & 0 & 0 & 0 \\ 0 & 0 & -3.170 & 0 \\ 0 & 0 & 0 & -0.150 \end{bmatrix} \begin{bmatrix} p \\ \phi \\ r \\ \beta \end{bmatrix} + \begin{bmatrix} -14.4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -0.96 \\ 0 & 0.037 & 0 \end{bmatrix} \begin{bmatrix} \delta_{a_c} \\ \delta_{r_c} \\ \delta_{p_c} \end{bmatrix} \tag{6}$$

$$\text{Characteristic equation} = s(s + 0.150)(s + 0.272)(s + 3.170) \tag{7}$$

A comparison of these results leads to the following conclusions:

The equations of motion and eigenvalues of the response-feedback decoupled system using the exact gains for model matching are those of the desired model.

The value of N_r' and corresponding eigenvalue for the response-feedback decoupled system using the approximate gains is -0.120 and differs from that of the model. The value of this parameter is dictated by the δ_p/β feedback gain. In theory, a value of $\delta_p/\beta = -72.771$ is required to achieve a value of $N_r' = -0.27$ for the closed-loop system. In Ref. 2 it is shown that

$$\delta_p/\beta = (N'_{r_m} - N'_r)/N'_{\delta_p} + (Y_r - 1)N'_{\delta_r}/Y_{\delta_r}N'_{\delta_p} \tag{8}$$

and using the rounded off value of $\delta_p/\beta = -73.0$ leads to a value of $N'_{rm} = -0.120$.

The eigenvalues for the geometric-decoupled system are the same as those of the model. However, the resultant values of L'_p and N'_r of the closed-loop system are interchanged with respect to those of the model. This fact accounts for the difference in the values of the δ_p/β and δ_a/β gains noted in Ref. 1.

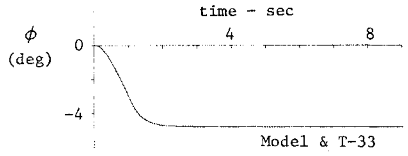

Fig. 2a ϕ response to pulse aileron command—response-feedback systems.

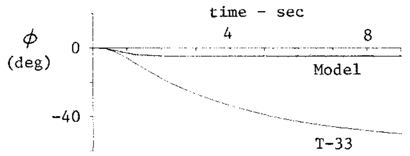

Fig. 2b ϕ response to pulse aileron command—geometric-decoupled system.

In order to further demonstrate the similarities and differences between the three decoupled systems being considered, time history responses for each are presented. As shown in Fig. 1, the β responses to a step rudder command for all three systems are identical to that of the model.

The roll responses, however, differ for the response-feedback and geometric-decoupled systems due to the different resultant values for L'_p. Fig. 2a shows the ϕ response to a pulse aileron command for the response-feedback systems; both exact and approximate. The model and T-33 responses are identical. A similar response for the geometric-decoupled system is shown in Fig. 2b. Note that the ϕ response for the same command is much larger and has a considerably longer rise time than that of the model due to the smaller resultant value of L'_p. This system is less acceptable from the flying qualities point of view since the roll mode response is too sluggish.

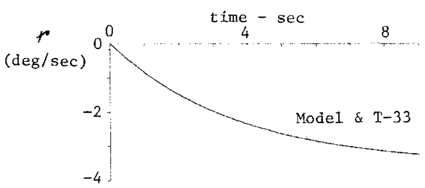

Fig. 3a r response to step drag petal command—response-feedback exact gain system.

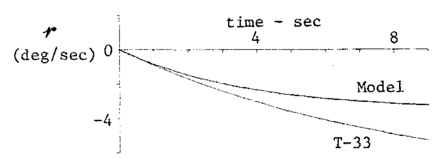

Fig. 3b r response to step drag petal command—response-feedback approximate gain system.

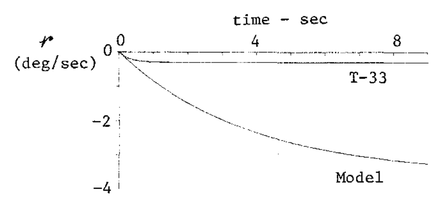

Fig. 3c r response to step drag petal command—geometric-decoupled system.

The yaw rate responses also differ due to the different values of the augmented N'_r obtained. Figure 3a shows the yaw response to a drag petal command for the response-feedback exact gain system. The model and T-33 responses overlay exactly. The corresponding variable is presented in Fig. 3b for the approximate response-feedback system and in Fig. 3c for the geometric-decoupled solution.

The reason the geometric-decoupling approach did not result in the model being duplicated is that the desired solution is not completely specified and the computer arbitrarily selects one of the three possible solutions. More specifically none of the real zeros are specified and only one of the desired transfer function responses is obtained. This approach is considerably more prone to "imperfect response matching" than the response-feedback system which assures "perfect response matching." It is argued in Ref. 1 that the geometric-decoupling method is useful "to arrive at (or close to) specified handling qualities." We feel that letting the computer select the ultimate transfer function response is less desirable than prespecifying the desired handling qualities solutions and forcing the control system to provide them.

It is worthwhile to compare the two techniques from the computational standpoint. The geometric-decoupling technique uses a complex vector space approach and requires a costly iterative technique to determine the solution. In the response-feedback approach, the problem is completely specified which allows hand computation of the gains.

The statement in Ref. 1 that "perfect model following is not assured" by the response-feedback method is simply not true. In theory, as long as sufficient number of independent controllers are available "perfect response matching" is assured. If this requirement is not met, no computation approach is capable of providing "perfect response matching." Within the above constraint, we can see no situation where "perfect response matching" is not assured.

Reference 1 indicates that in the response-feedback method, "if the tangent of the reference pitch angle is not zero ($\dot{\phi} \neq p$) the method fails." The assumption that $\dot{\phi} = p$ is included in Ref. 2 because it is indeed valid for the problem presented. If this assumption is not valid, one simply does not include it in developing the equations for the simulator vehicle and perfect response matching is still achieved via the response-feedback technique. It should be pointed out that "perfect response matching" can be achieved for a model in which $\dot{\phi} \neq p$ even when the assumption that $\dot{\phi} = p$ is valid for the simulator vehicle.

We conclude that the Cliff/Lutze geometric-decoupling approach will indeed provide decoupled lateral-directional responses but that without further specifying the problem, i.e., the complete transfer function numerator characteristics, there is no guarantee that the desired handling qualities solution will be obtained. When the problem is completely specified, the solution is unique and the computational complexity introduced by their omission is removed.

References

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